

C.M.1974/C.19

On Conditions and Reasons of Formation  
of the Oxygen Minimum Layer in the Ocean

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Abstract. The paper analyses a solution of a differential equation of turbulent diffusion including biochemical consumption of oxygen and advection with different values of initial parameters and boundary conditions. It shows impossibility of formation of the oxygen minimum layer when biochemical consumption of oxygen (BCO) is absent, influence of thickness of the photosynthesis layer on a depth of occurrence of the oxygen minimum layer and significance of advection in the vertical distribution of the dissolved oxygen content.

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A great number of papers deal with conditions and reasons of formation of the oxygen minimum layer in the ocean and those by Saywell (1937), Sverdrup (1938), Wattenberg (1938), Smetnin (1959), Virtky (1962), Skopintsev (1965) and Bubnov (1967, 1970) are worthy of notice. In spite of some differences in estimation of significance of particular factors forming the oxygen minimum layer majority of authors believe that its formation resulted from certain combinations and interaction of biochemical consumption of oxygen, vertical turbulent

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exchange and horizontal advection.

The given paper is aimed at a quantitative estimation of some factors resulting in formation of the oxygen minimum layer.

In the general case for description of the distribution of dissolved oxygen in the ocean as a function of Cartesian coordinates and time the following equation can be used:

$$\frac{\partial k}{\partial t} + u \frac{\partial k}{\partial x} + v \frac{\partial k}{\partial y} + w \frac{\partial k}{\partial z} = \frac{\partial}{\partial x} \left( A_x \frac{\partial k}{\partial x} \right) + \frac{\partial}{\partial y} \left( A_y \frac{\partial k}{\partial y} \right) + \frac{\partial}{\partial z} \left( A_z \frac{\partial k}{\partial z} \right) + Q_1 + Q_2 \quad (1)$$

where  $u$ ,  $v$  and  $w$  are constants of the current velocity along coordinate axes (axis  $OZ$  is directed downwards),

$k$  - concentration of dissolved oxygen in ml/l,

$A_x$ ,  $A_y$ ,  $A_z$  - coefficients of the turbulent exchange in the direction of coordinate axes,

$Q_1$  - release of oxygen in unit volume per unit time (for instance, owing to photosynthesis; considering the water layer to be deeper than the photosynthesis one, we assume in the first approximation that  $Q_1 = 0$ ),

$Q_2$  - loss of oxygen in unit volume per unit time owing to the biochemical consumption of oxygen (BCO).

When axis  $OX$  runs along the middle direction of the current in some layer one can take that in this layer:

$$v = 0 \text{ and } v \frac{\partial k}{\partial y} = 0 \quad (2)$$

According to Kolesnikov (1961) we assume that coefficient  $A_z$  indicates the total action of a vertical turbulent exchange and vertical flows, then one can take that

$$w \frac{\partial k}{\partial z} = 0$$

Let us compare an order of terms characterizing turbulent diffusion in different directions:

$$\begin{aligned} O\left(\frac{\partial}{\partial x} A_x \frac{\partial K}{\partial x}\right) &= O\left(\bar{A}_x \frac{\partial^2 K}{\partial x^2}\right) = 10^6 \text{ cm}^2 \text{ sec}^{-1} \cdot 1 \text{ ml/l} / \\ &= 10\,000 \text{ km}^2 = 10^{-12} \text{ ml/l sec} \\ O\left(\frac{\partial}{\partial y} A_y \frac{\partial K}{\partial y}\right) &= O\left(\bar{A}_y \frac{\partial^2 K}{\partial y^2}\right) = 10^6 \text{ cm}^2 \text{ sec}^{-1} \cdot 1 \text{ ml/l} / 1000 \text{ km}^2 = \\ &= 10^{-10} \text{ ml/l sec} \\ O\left(\frac{\partial}{\partial z} A_z \frac{\partial K}{\partial z}\right) &= O\left(\bar{A}_z \frac{\partial^2 K}{\partial z^2}\right) = 10^2 \text{ cm}^2 \text{ sec}^{-1} \cdot 1 \text{ ml/l} / 1 \text{ km}^2 = \\ &= 10^{-8} \text{ ml/l sec} \end{aligned}$$

From this it follows that a horizontal turbulent exchange is less essential in formation of the oxygen field than a vertical one and terms having expressions  $A_x \frac{\partial K}{\partial x}$  and  $A_y \frac{\partial K}{\partial y}$  in equation (1) can be ignored.

Then with a stationary process ( $\frac{\partial K}{\partial t} = 0$ ) equation (1) can be written as:

$$\frac{\partial}{\partial z} A_z \frac{\partial K}{\partial z} + Q_z \quad (4)$$

For the tropical zone of North Atlantic we (Rossov 1967) have obtained

$$A_z = a + bz^c \quad (5)$$

where values  $a$ ,  $b$  and  $c$  are ~~now~~ being determined through a vertical distribution of stability. Variations of these values (when dimensionality of  $A_z$  in  $\text{km}^2/\text{sec}$  that is more convenient for calculating) were  $a = (-1 \div 1) \cdot 10^{-10}$ ;  $b = (0.07 \div 0.12) \cdot 10^{-8}$ ;  $c = 1.1 \div 1.9$ .

Having used the method of determination of integral value  $A_z$  again (in the sense of the common calculation of  $A_z$  and  $w$  (Ko-

Kolesnikov 1961) through stability for the area of the beginning of the North-Atlantic Current <sup>x)</sup> we have seen that for this region ratio (5) is true; here it was written in the form of:

$$A_z = (0.014 + 0.385 z^{1.33}) 10^{-8} \frac{\text{km}^2}{\text{sec}} \quad (6)$$

where  $z$  - in km.

Change in biochemical consumption of oxygen with depth was studied by a number of authors; Fig.1 shows function  $Q_2(z)$  below the photosynthesis layer obtained according to different estimations. In this paper there is given a ratio for waters below the photosynthesis layer used before (Rossov 1967):

$$Q_2(z) = a_0 K/z \quad (7)$$

where  $z$  - depth in km and  $a = 10^{-9}$  km/sec.

This function is convenient to calculate and as it is evident from Fig.1 it has values close to average ones comparing to the other functions.

In the first approximation we assume that

$$u \frac{\partial K}{\partial z} = a_1 + b_1 z \quad (8)$$

Taking into account the said above, equation (4) will be written as:

$$a_1 + b_1 z = (a + bz^c) \frac{\partial^2 k}{\partial z^2} + bc z^{c-1} \frac{\partial k}{\partial z} + \frac{a_0 k}{z} \quad (9)$$

Otherwise, when no advection and  $a=0$  and  $c=1$ , equation (9)

x)

Stability  $E_0$  was calculated by V.V.Burmakin, passing from stability to  $A_z(T)$  was made by averaged formula  $A_z(T) = 7 + \frac{6.6 \cdot 10^{11}}{z}$  (Kolesnikov 1961) (6a). When passing from  $A_z(T)$  to  $A_z(K)$   $E_0$  there was assumed again that  $A_z(K)$  is one order less than  $A_z(T)$  (Rossov 1967).

can be written in the form of:

$$bzk'' + bk' - \frac{a_0 k}{z} = 0 \quad (10)$$

When no biochemical consumption of oxygen (BCO) ( $a_0 = 0$ ) we have:

$$bzk'' + bk' = 0 \quad (10a)$$

that is equivalent to:

$$(zk')' = 0 \quad (10b)$$

Hence:  $k = c_1 \ln z + c_2 \quad (11)$

Evidently function (11) has no extremes because the sign of derivative  $k' = \frac{c_1}{z}$  does not change with any values of  $z > 0$ . Consequently, formation of the oxygen minimum layer with any values of the coefficient of a vertical turbulent exchange (conditions:  $a=0, c=0$  do not limit values  $b$  and  $A$ ) is impossible in case BCO does not occur or: presence of BCO is a necessary condition for existence of the oxygen minimum layer (the problem on sufficiency of this condition will be discussed below).

The general integral of equation (10) can be calculated if we take  $k = z^n$ , then after elementary transformations we obtain:

$$k = c_1 z^{\sqrt{\frac{a_0}{b}}} + c_2 z^{-\sqrt{\frac{a_0}{b}}} \quad (12)$$

where:  $c_1$  and  $c_2$  are calculated from boundary conditions.

Proceeding from Fig.1 and values  $a$  and  $b$  given above in equation (5) one can take that

$$\begin{aligned} a_0 &= (5 \div 15)10^{-10} \\ b &= (5 \div 60)10^{-10} \end{aligned} \quad (13)$$

consequently,  $0.08 \leq \frac{a_0}{b} \leq 3$ .

Fig.2 depicts graphs of function (12) constructed for

different values of  $\frac{a_0}{b}$  with given boundary conditions (values  $k(z)$  for some superior boundary where influence of photosynthesis is quite low and for the inferior boundary where  $k(z)$  has the maximum value). Fig. 2 makes it possible to conclude that the value of the oxygen content in the oxygen minimum layer is determined by the value of ratio  $\frac{a_0}{b}$ ; the greater the ratio the more clearly defined the oxygen minimum layer but with low values of  $\frac{a_0}{b}$  it cannot form at all. It is noteworthy that the depth of occurrence of the oxygen minimum layer does not almost depend on value  $\frac{a_0}{b}$  or the position of the lower boundary of the layer calculated; but a change in a position of the lower boundary of the photosynthesis layer sharply changes a position of the oxygen minimum layer. This conclusion particularly explains increase in depth of the oxygen minimum layer in the Gulf Stream eastwards: increase in transparency of waters of the Gulf Stream proper, compared to coastal waters, leads to increase of thickness of the photosynthesis layer and due to that the oxygen minimum layer sinks.

It is the increase in thickness of the photosynthesis layer and not displacement deeper and eastwards of "cores" of sunk undersaturated waters with oxygen waters, as Adrov (1971) believes, is one of the principal causes of increase in depth of the oxygen minimum layer occurrence beneath the Gulf Stream in the eastern direction.

Equation (10) does not show significance of an advective factor which is, as it will be given below, also affects a position and intensity of the oxygen minimum layer; however, an analysis of the simplest cases given above already shows the principal regularities in formation of the oxygen minimum layer:

necessity of existence of biochemical consumption of oxygen and sufficiency of a certain combination of the value of BCO and a vertical turbulent exchange.

Estimation of influence of a lateral turbulent exchange is not complicated.

Taking in the first approximation that

$$A_y \frac{\partial^2 k}{\partial y^2} = m \quad (14)$$

we obtain a differential equation written in the form of:

$$bzk'' + bk' - \frac{a_0 k}{z} + m = 0 \quad (15)$$

Its particular solution must be in the form of:

$$k_1 = nz \quad (16)$$

Substituting  $k_1 = nz$  in (15) we find  $n$  and obtain

$$k_1 = \frac{mz}{a_0 - b} \quad (17)$$

Thus, the general solution of equation (15) will be:

$$k = \frac{mz}{a_0 - b} + c_1 z^{\sqrt{\frac{a_0}{b}}} + c_2 z^{-\sqrt{\frac{a_0}{b}}} \quad (18)$$

In case as it is given above  $[O(m)] = 10^{-10}$ ,  $a_0 = (5 \div 15) \cdot 10^{-10}$ ,  $b = (5 \div 60) \cdot 10^{-10}$  then one can see that value of term  $\frac{mz}{a_0 - b}$

is determined by  $a_0 - b$ , though in the general case (with typical values:  $a_0 = 10 \cdot 10^{-10}$ ,  $b = 20 \cdot 10^{-10}$ ) this term is considerably less than the sum of the rest terms in the right part of expression (18).

Now let's analyse a solution of the more complete and general equation which provides for a vertical turbulent exchange (without simplified assumptions), BCO and horizontal advection:

$$(a+bz^c)k'' + bcz^{c-1}k' + \frac{a_0 k}{z} + b_1 z + a = 0 \quad (19)$$

with different values of initial parameters  $a$ ,  $b$ ,  $c$ ,  $a_0$ ,  $a_1$ ,  $b_1$ . It should be noted that with an appropriate value of  $a_1$  a horizontal turbulent exchange can be provided for (if it equals to a constant value).

Solution of equation (19) was made by a computer by A. Savateeva. The boundary conditions were:  $K(0.2 \text{ km}) = 5.0 \text{ ml/l}$ ,  $K(2.2 \text{ km}) = 6 \text{ ml/l}$ . The calculation was carried out with step  $z = 0.2 \text{ km}$ .

The following combinations of values of initial parameters (dimensionality in km, common factor  $10^{-10}$  is dropped in all cases):

1.  $a_0 = -10$ ;  $c = 1.5$ ;  $b = 10$ ;  $a_1 = b_1 = 0$

$a = 0.0$ ;  $0.5$ ;  $1.0$ ;  $3.0$

2.  $a_0 = -10$ ;  $c = 1.5$ ;  $a = a_1 = b_1 = 0$

$b = 5$ ;  $10$ ;  $15$ ;  $30$ ;  $50$

3.  $a_0 = -10$ ;  $b = 10$ ;  $a = a_1 = b_1 = 0$

$c = 0.7$ ;  $1.3$ ;  $1.6$ ;  $2.2$

4.  $b = 10$ ;  $c = 1.5$ ;  $a = a_1 = b_1 = 0$

$a_0 = 0.0$ ;  $-5.0$ ;  $-7.5$ ;  $-10.0$ ;  $-12.5$ ;  $-15.0$

Different variations of a change of the vertical turbulent exchange coefficient with depth with mean values of BCO; no advection

Different values of BCO with mean values of  $A_z$ . No advection

The principal results of calculation are given in Fig. 3 which show that great values of  $A_z$  are conformed with weak expression of the oxygen minimum layer and vice versa, great values of BCO are agreed with a low content of oxygen in the minimum layer. Values  $a$  and  $c$  affect a depth of occurrence of the oxygen minimum layer to a greater extent than the rest parameters, however variations of this depth are not great and occur in reality much more rarely.

To take into account a role of advection we shall proceed



from the following assumptions: we take that  $\rho = 1 \text{ ml/10 000 km} = 10^{-4} \text{ ml/km}$ ; we also assume that velocity of the current changes with depth according to the linear law. Solution of equation (19) was made by a computer with  $a = 0$ ,  $a_0 = -10$ ,  $b = 10$ ,  $c = 1.5$  for the combinations of values of  $v_0$  (velocity of the surface current) and  $z_0$  (depth of occurrence of the O-surface) and boundary conditions given (Table 1).

Fig.4 gives the main results of calculation. While considering Fig.4 one can conclude:

a) in all cases there is coincidence of a position of the O-surface and a point of curve bend  $k(z)$ ;

b) when the O-surface occurs at a small depth and considerable advection of oxygen by deep waters takes place (Curve 1 in Fig.4b) the oxygen minimum layer can be absent;

c) when a position of the O-surface is constant but velocities of the current are different, the depth of occurrence of the oxygen minimum layer does not change and only the oxygen content changes in this layer;

d) even significant changes in a position of the O-surface do not cause essential changes in a position of the oxygen minimum layer.

Table 1

$k(0,2) = 5 \text{ ml/l}$

$k(2,2) = 6 \text{ ml/l}$

$V_0$	cm/sec	30	20	10	5	20	20	20	20	20	10	10	10	10	10
$Z_0$	km	1	1	1	1	4	1,67	1,33	0,66	0,5	2	1,33	0,60	0,5	0,33

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Fig. 1. Vertical distribution of BCO by data according to different authors: 1 - according to Saywell (1937), 2 - 2a - to Skopintsev (1965) under different conditions, 3 - according to Bubnov (1967) for 50°N, 4 - to Rossov (1967).

Fig. 2. Vertical distribution of the content of dissolved oxygen with different values of ratio  $a_o/b$  in equation (12):

- |  |   |
|--|---|
| $\left. \begin{array}{l} 1. a_o/b=9, \\ 2. a_o/b=3, \\ 3. a_o/b=2, \\ 4. a_o/b=1, \\ 5. a_o/b=0.5, \\ 6. a_o/b=0.25 \\ 7. a_o/b=0.08 \end{array} \right\} \begin{array}{l} K(0.2 \text{ km})= \\ =5 \text{ ml/l} \\ K(2.2 \text{ km})= \\ =6 \text{ ml/l} \end{array}$ | $\begin{array}{l} 8. a_o/b=2; K(0.2 \text{ km})=5 \text{ ml/l}; \\ \quad K(1 \text{ km})=6 \text{ ml/l} \\ 9. a_o/b=2; K(0.4 \text{ km})=5 \text{ ml/l}; \\ \quad K(3 \text{ km})=6 \text{ ml/l} \\ 10. a_o/b=2; K(0.2 \text{ km})=5 \text{ ml/l}; \\ \quad K(3 \text{ km})=6 \text{ ml/l} \end{array}$ |
|--|---|

Fig. 3. Vertical distribution of the content of dissolved oxygen with different values of initial parameters in equation (19) when no advection:

- |   |  |
|---|--|
| $\begin{array}{l} a. \left. \begin{array}{l} 1. a = 3 \\ 2. a = 0 \\ 3. b = 50 \\ 4. b = 5 \\ 5. c = 0.7 \\ 6. c = 2.2 \end{array} \right\} \begin{array}{l} a_o = -10; c=1.5; b=10 \\ a_o = -10; c=1.5; a=0 \\ a_o = -10; b=10; a=0 \end{array} \end{array}$ | $\begin{array}{l} b. \left. \begin{array}{l} 1. a_o=0 \\ 2. a_o=-5 \\ 3. a_o=-10 \\ 4. a_o=-15 \end{array} \right\} \begin{array}{l} b=10 \\ c=1.5 \\ a=0 \end{array} \end{array}$ |
|---|--|

Fig. 4. Vertical distribution of the content of dissolved oxygen

when advection occurs and with different values of  $v_0$  and  $z_0$ :

- a. 1.  $v_0 = 5$  cm/sec  
2.  $v_0 = 10$  cm/sec  
3.  $v_0 = 20$  cm/sec  
4.  $v_0 = 30$  cm/sec
- }  $z_0 = 1$  km

- b. 1.  $v_0 = 10$  cm/sec;  $z_0 = 0.33$  km  
2.  $v_0 = 20$  cm/sec;  $z_0 = 0.67$  km  
3.  $z_0 = 0.5$  km  
4.  $z_0 = 0.67$  km  
5.  $z_0 = 1.33$  km  
6.  $z_0 = 2.0$  km
- }  $v_0 = 10$  cm/sec

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